Vassiliev invariants of Legendrian, Pseudo-Legendrian and framed knots in contact 3-manifolds

ABSTRACT: A cooriented contact structure on a 3-manifold M is a two-plane distribution C in TM that is the kernel of a one-form α with $\alpha \wedge d\alpha \neq 0$. A knot in (M, C) is Legendrian if it is everywhere tangent to C. Clearly every Legendrian knot has a natural framing.

We show that for a big class of contact manifolds (M, C) the groups of order $\leq n$ invariants (with values in an arbitrary Abelian group) of Legendrian, and of framed knots are canonically isomorphic. This class of contact 3-manifolds includes all tight contact (M, C), all hyperbolic M, and all (M, C) such that the Euler class of C is in the torsion of $H^2(M)$. As a corollary we get that for all surfaces F the group of finite order Arnold's J^+-type invariants of wave fronts on F is canonically isomoprhic to the group of Vassiliev invaraints of framed knots in the spherical tanagent bundle STF of F.

On the other hand we show that in many examples the groups of Gousarov-Vassiliev invariants of Legendrian and of framed knots are different and Vassiliev invariants of Legendrian knots can distinguish Legendrian knots that are isotopic as framed knots.

Let V_C be a cooriented vector field of a contact structure C. A knot in (M, V_C) is Pseudo-Legendrian if it is nowhere tangent to C. We show that for all contact three manifolds the groups of Gousarov-Vassiliev invariants of Legendrian and Pseudo-Legendrian knots are canonically isomorphic.